# RECITATION 8

## RELATED RATES, AND LINEAR APPROXIMATIONS

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## *Section* 1. *Exercises*

**Exercise 1**

Suppose a writer is writing a book, and has a level of enjoyment given by $E(p) = 5p^2 - e^p$ for $p$ pages written. Suppose the writer always writes 5 pages per day, but has thus far written 0 pages. Calculate the rate of change in the writer's level of enjoyment.

*Solution* .:.

Using the chain rule or implicit differentiation, $\frac{dE}{dt} = \frac{dE}{dp}\frac{dp}{dt} = (10p - e^p) \cdot 5$. For $p = 0$, we get $\frac{dE}{dt} = (0-1) \cdot 5 = -5$.

**Exercise 2**

Suppose $y^3 + 2xy^2 = 8$. Suppose $x = 0$ and $\frac{dx}{dt} = 2$. Calculate $\frac{dy}{dt}$.

*Solution* .:.

Using implicit differentiation, $3y^2\frac{dy}{dt} + 2\frac{dx}{dt}y^2 + 4y\frac{dy}{dt}x = 0$. Since $\frac{dx}{dt} = 2$ and $x = 0$, this tells us $3y^2\frac{dy}{dt} + 4y^2 = 0$, or that $\frac{dy}{dt} = -4/3$ assuming $y \neq 0$. But $y \neq 0$ since $x = 0$: $y^3 + 0 = 8$ implies $y = 2 \neq 0$.

**Exercise 3**

The cost of making something is given by $C(t) = 4t^2 - 10t + 80$ for $t$ measured in years. Estimate the cost six months after $t = 0$.

*Solution* .:.

We are estimating $C(.5)$. Using a linear approximation, $C(.5) \approx C(0) + C'(0) \cdot .5$ so that the new cost is approximately $80 + (8 \cdot 0 - 10) \cdot (.5) = 75$.

**Exercise 4**

Suppose $x^2 + y^2 = 13^2$ with $y = 12$. Suppose $\frac{dy}{dt} = -5$ and $x > 0$. Calculate $\frac{dx}{dt}$.

*Solution* .:.

Implicit differentiation yields that $2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$, i.e. that $\frac{dx}{dt} = -\frac{y}{x}\frac{dy}{dt}$. Since $\frac{dy}{dt} = -5$ and $y = 12$, $\frac{dx}{dt} = 60/x$ where $x^2 + 12^2 = 13^2$. We can calculate that then $x = \sqrt{13^2 - 12^2} = \sqrt{169 - 144} = \sqrt{25} = 5$. Therefore $\frac{dx}{dt} = 60/5 = 12$.

**Exercise 5**

For $f(x) = x^3 + x$, approximate $f(.9)$.

*Solution .:.*

$f(.9) \approx f(1) + f'(1)(.9-1) = f(1) + f'(1)\cdot(-.1)$. We have that $f'(x) = 3x^2 + 1$ so that $f'(1) = 4$. Moreover, $f(1) = 2$ so that $f(.9) \approx 2 + 4(-.1) = 2 - .4 = 1.6$.

**Exercise 6**

The cost of making $x$ things is $C(x) = 2x^2 + e^{x-9}$.

1. Using marginal analysis, estimate the cost of making the 10th thing.
2. Suppose each thing sells for $S(x) = 3x - 1$ dollars when there are $x$ made. Estimate the revenue from the 10th thing.

*Solution .:.*

The marginal cost of making 10 things is given by $C'(9)$ since $C(x) \approx C(9) + C'(9)\cdot(x-9)$ has $C(10) \approx C(9) + C'(9)$. In particular, the cost of the 10th item is $C(10) - C(9) \approx C'(9)$. We have that $C'(x) = 4x + e^{x-9}$ so that $C'(9) = 4\cdot 9 + e^0 = 36 + 1 = 37$.

The revenue is given by $R(x) = x\cdot S(x) = 3x^2 - x$. Thus $R'(x) = 6x - 1$. Therefore $R(10) \approx R(9) + R'(9)(10-9) = R(9) + R'(9)$ yields $R(10) - R(9) \approx R'(9)$. Therefore the revenue from the 10th thing is given by $R'(9) = 54 - 1 = 53$.